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DESIGNING HEAT EXCHANGER NETWORKS FOR EXISTING CHEMICAL PLANTS

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Abstract—New targets for the heat exchanger network synthesis problem, which account for both heating and power needs, are defined. Especially useful when updating existing facilities, the proposed targets stand for both the maximum heat flow to be recovered and the types and number of heating utilities to be allocated if all plant energy requirements are to be satisfied at the lowest fuel cost. The new goals are computed by solving a small-scale nonlinear programming problem introduced in this paper. Accounting for them, a heat exchanger network involving a low number of units is synthesized using the transportation formulation[1]. The results clearly show the economic disadvantages of recycling process heat beyond a certain limiting value, unless bottlenecks in the current steam-power system, precluding further fuel savings, are removed.

Scope—The synthesis of heat exchanger networks is a well-studied problem whose primary goal is to find the optimal network design. In this problem, a process stream that gives heat is called a hot stream, whereas a cold stream is one that demands heat. A network design is said to be feasible when all hot and cold streams meet process requirements through either recycling heat from hot to cold streams or using heating/cooling utilities. Each network is characterized not only by the overall utility usage, but also by the amount of each kind demanded. During the last decade many synthesis procedures have been proposed, most of which choose two targets for the network design: (i) minimum utility usage; (ii) minimum number of heat exchangers [2,3]. Such targets can be predicted beforehand by using systematic procedures like the problem table [3] or the transportation model [1]. In their review article, Nishida *et al.* [2] pointed out that results available in the area are already being applied in industry, leading to significant savings in both operating and fixed costs.

However, most of the current synthesis methods have been developed by studying only the process, i.e. the energy-consuming system, without considering the energy supply system structure. This could be misleading because the ultimate goal of any energy conservation program is aimed at minimizing fuel consumption in the energy supply system [4]. Since steam produced at boilers is first used to generate power in backpressure turbines and then allocated to meet a large portion of heating requirements, simultaneous consideration of both process heating and power demands becomes necessary. In this way two new targets for the network design are derived so as to meet heat and power needs at the lowest fuel consumption. They are (i) the economic utility usage and (ii) the optimal utility allocation.

A nonlinear mathematical programming problem is proposed to predict both targets and is solved by using the Successive Linear Programming method [5]. The method consists in solving a finite sequence of linear programming problems, each one obtained by replacing the nonlinear constraints by first-order Taylor expansions around the current solution. The limit of the optimal solution sequence generally provides the program global optimum. A good starting point is obtained by an initialization procedure that accounts for the optimal solution properties. With the values of the new targets established, a slight modification of the transportation-based method proposed by Cerdá and Westerberg [1] allows the heat exchanger network to be synthesized.

To illustrate the technique, an example problem is solved: An energy conservation project is undertaken in an existing chemical plant to reduce fuel through cost-effective heat integration. The current structure of the steam-power plant is assumed to remain unchanged. Results are compared with those that would have been obtained if present targets had been used.

Conclusions and Significance—Previous works have shown that the present process design creates a bottleneck that limits heat integration possibilities. Such a limit is measured through the minimum utility usage, adopted by many authors as one of the heat exchanger network targets. Noting that the real network goal is the least fuel consumption, this work reveals that the current configuration of the steam-power plant usually introduces a secondary bottleneck that imposes a still lower bound on the economic benefits of a heat integration project.

On the basis of the minimum fuel consumption goal, two new targets for the network design are identified: (1) the economic process utility usage; (2) the optimal heating utility allocation. The former target comes from the fact that steam is also required in a chemical plant to generate power. Steam flows coming out of backpressure turbines should be mostly allocated for heating purposes. If steam flows available owing to power needs are higher than steam requirements at pinch conditions, there is no economic reason to lower heat demands beyond what is called economic utility usage. With the new goal, low- and medium-level utility pinches, which usually appear in the optimal network design obtained through current procedures [6], are avoided. This leads to a smaller number of heat exchangers in the network design and a lower overall heat transfer area, without increasing fuel consumption at boilers.

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Moreover, it is usually assumed in the literature that the utilities available for allocation in the network are given and their amounts unlimited. This paper shows that the quality of a network measured by the plant fuel consumption strongly depends on the amounts of each kind of utility allocated in the process. Thus the concept of optimal utility allocation is introduced as the one that decreases plant fuel consumption as much as possible by maximizing the steam Rankine cycle efficiency. Before using an HX-network synthesis algorithm, the optimal utility allocation should be found.

The values of the new targets are obtained by solving a nonlinear mathematical programming problem through the Successive Linear Programming technique. The computational algorithm consists in solving a finite sequence of linear programming problems by using the Simplex method. Convergence of the procedure is sharply improved by a good starting point provided by a simple initialization scheme. An example problem is solved, and the results show a large difference between the minimum and economic heat recovery targets, unveiling the need for changes in the present steam-power system. Then, a transportation formulation is used to synthesize a network design that is able to get the lowest fuel consumption with a small number of heat exchangers.

1. INTRODUCTION

Rising fuel prices have adversely affected operating costs of energy-intensive industries like oil refineries and petrochemical plants. This shift in the economic balance between capital and running costs has created real incentives for finding new ways of reducing energy consumption in chemical processes. Since 1968 a significant number of papers providing a variety of systematic approaches toward helping design engineers identify cost-effective process modifications have been published. One area of energy conservation receiving increased attention is improved process heat recovery. In their review, Nishida *et al.* [2] point out that available results in this area are already being applied in industry, leading to significant savings in both operating and fixed costs. However, especially in the revamping of existing petrochemical facilities, they suggest recovering process heat beyond what an overall economic analysis would recommend. The shortcomings of current procedures come from the fact that they were developed by analyzing the process, exclusively, i.e. the energy-consuming system, disregarding the energy supply system structure. Since the ultimate goal of any energy conservation program is aimed at minimzing fuel consumption in the energy supply sysem, it becomes clear that current algorithms can yield misleading results because part of the necessary information is ignored [7]. This approximation, however, works quite well in industrial facilities like oil refineries, where heat supplied to the process comes mostly from fired heaters. Then, any heat recovered and reused in the process produces an equivalent decline in the plant fuel consumption. A different situation may arise when steam generated at boilers to first produce power in backpressure turbines is then allocated to meet a large portion of the heating requirements.

In consideration of the lowest cost steam-power-system design problem, several optimization procedures have already been proposed and used in industry for either on-line or off-line purposes. In both cases the problem is modeled as a mixed-integer nonlinear mathematical program, and its solution provides the optimal structure for performing the required functions at minimum cost [4,8,9]. For on-line applications, a flexible design is created by using steam turbines with spare electric drives. In doing so, load disturbances and variations in demands that shift the optimal steam-power-plant operation can be accommodated without any major loss in the steam Rankine-cycle efficiency.

On the other hand, the synthesis of heat and power networks, where the recovery and use of heat and mechanical energy are simultaneously considered, has quite recently begun to receive special attention. Studies due to Nishio *et al.* [10] have shed new light on the area. They introduce a systematic approach to achieve optimal use of steam and power in a chemical plant through the appropriate energy coordination between processes and the steam-power system. The energy coordination problem is described by an LP problem with demand-dependent coefficients whose general feasible solution is found in terms of process heat and power requirements. The synthesized optimal structure for the heat and power supply system allows process requirements to be met at minimum fuel consumption in furnaces and boilers. To enlarge the problem solution space, several alternatives are considered for both heating and power generation. In turn, Linnhoff and Townsend [11] use the widely known concept of *pinch point* to derive criteria for the appropriate allocation of gas or steam turbines and heat pumps in process networks. By doing so, 100% of the energy supplied to process systems can be transformed into heat and power needs.

This paper primarily deals with the search for a heat exchanger network design for an existing process, accounting for the current structure of the steam-power facilities. Before designing the heat exchanger network, one needs to know the extent to which heat recovery is economically justified, as well as the kinds and amounts of utilities to allocate in the process to meet energy demands at the minimum fuel cost. Second, it intends to provide a way to unveil bottlenecks in the current steam-power-plant configuration that prevent further energy savings from being achieved.

2. BRIEF DESCRIPTION OF A STEAM POWER PLANT

A simplified typical steam-power plant in a petrochemical complex is shown in Fig. 1. A combined heat and power scheme is used where a combustion turbine/

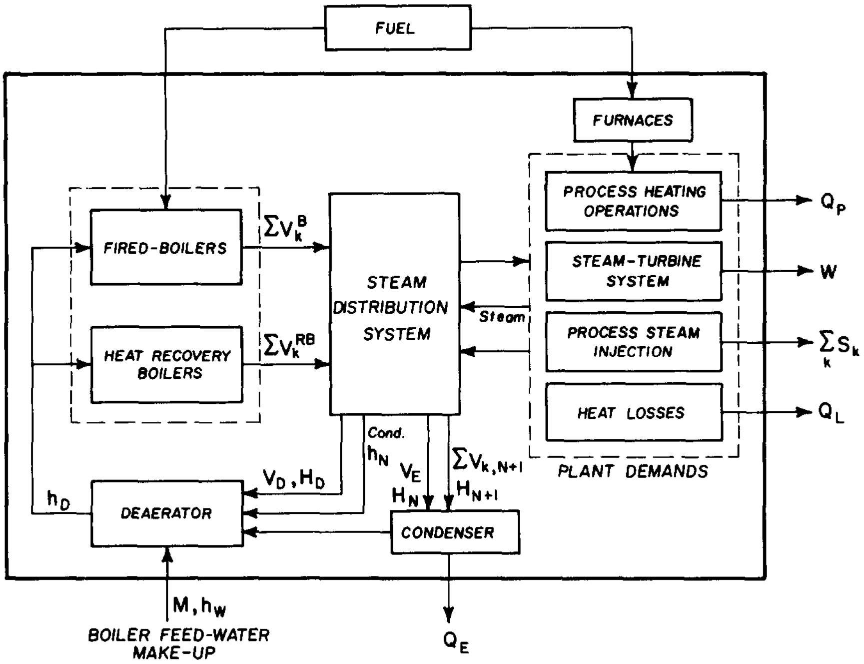

Fig. 1. Simplified diagram of the steam system in a petrochemical complex.

alternator provides electric power and the energy still available in the hot exhaust gas is used in heat recovery boilers with or without supplementary firing. In addition, independently fired boilers can also be included in the steam plant where high-pressure superheated steam is produced both to generate mechanical power and supply heat to plant users. In a chemical industry, the production of steam is aimed at fulfilling different purposes: (i) for process streams heating; (ii) for driving pumps, fans, blowers, compressors and often electric generators through backpressure and condensing turbines; (iii) for use in steam-cracking units, strippers or steam ejectors.

Generally, a large part of the process heating demands can be met by using medium-pressure (approximately 30 bars, 480°K) or low-pressure (approximately 2.5 bars, 425°K) steam. This fact is very important for two reasons: (1) the value of the steam latent heat goes up as the pressure decreases; (2) a pressure drop from around 60 bars, at which the boilers are operated, to 30 or 2.5 bars is available to produce power in backpressure turbines. When the power needs exceed the demands for process steam injection and heating, it becomes convenient to use condensing turbines that exhaust steam at a pressure below atmospheric. This procedure substantially cuts down the boiler steam production required to meet process heat and power demands. To make this scheme workable, a steam distribution system, usually comprising three steam headers operating at high, medium and low pressure, respectively (see Fig. 2), is provided. Each header collects steam coming from both the boilers and backpressure turbines at similar pressure and conveys it to process users (steam heaters, backpressure and condensing turbines, strippers, steam cracking units and so forth). To recover additional heat, fractions of high- and medium-pressure condensates leaving steam heaters are flashed to generate lower pressure steam, which returns to the steam circuit. The remaining condensate flows are recycled and used for desuperheating the steam flows going to steam heaters (see Fig. 2).

In turn, low-pressure condensate is supplied to the boiler feed water system, along with make-up water. Before that, it is circulated through the deaerator, where it is heated using low-pressure steam (see Fig. 1). Mass balance at each steam header is set by installing letdowns with or without desuperheating stations. The letdowns allow the steam to flow through an adjustable valve from a higher to a lower pressure header, thus losing the possibility of generating supplementary power.

Hereinafer, the overall heat and power requirements in a chemical plant or complex are called heat and power loads, respectively, while the rate of fuel burned in boilers, gas turbines and fired heaters to meet plant needs are referred to as the plant fuel load.

The fuel consumption in boilers is closely related to the overall steam flow supplied to the steam distribution system, usually called boiler load. The value of this variable depends primarily on

(1) size and temperature levels of process heat demand,
(2) heating utility allocation policy,
(3) power load,
(4) configuration of the power-generating system,
(5) number and operating pressures of steam headers.

With features (3)–(5) fixed, the process engineer

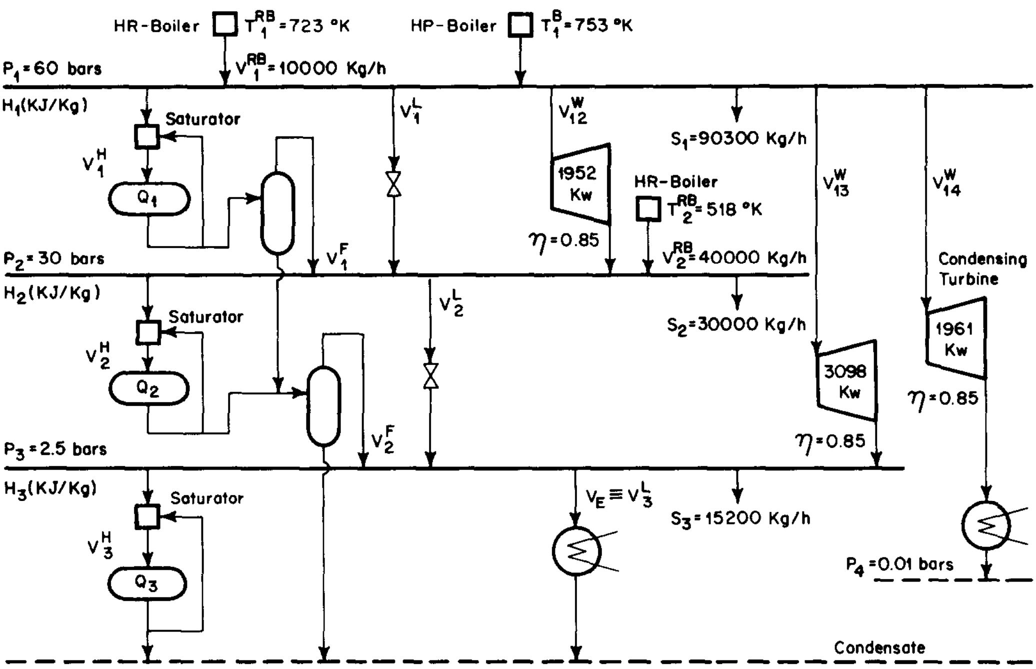

Fig. 2. Actual configuration of the complex steam distributing and power production systems for the example under study.

still has decision variables to manipulate to lower the chemical plant fuel load.

3. THE UTILITY ALLOCATION PROBLEM

Accounting for the temperature level at which it demands heat, each plant user is assigned to a particular steam header. Since high-, medium- and low-pressure steam headers are usually available at the steam plant, there generally exist several feasible options rather than a single one. But only some of them allow also the steam-power system to operate at the highest efficiency. They are called optimal utility allocation policies because through them process heat demands are satisfied at the lowest fuel cost. Such policies minimize both steam letdowns (i.e. steam pressure drops with no power production) and steam oversupply (i.e. low-pressure steam with no use in the plant). In Fig. 2, V_k^L ($k = 1, 2$) and V_E stand for steam letdowns and LP-steam oversupply, respectively.

Stream data for a six-stream HX-network synthesis problem are shown in Table 1. By means of the procedure proposed by Linnhoff and Flower [3], the problem is divided into seven temperature intervals corresponding to subnetworks. The intervals are defined by the stream supply temperatures, as shown in Table 2, where the net heat demands at the subnetworks are listed in the last column. The minimum utility usage and the pinch location can be found by using the problem table algorithm of Linnhoff and Flower [3]. The heat flow diagram is shown in Fig. 3(a). No process pinch occurs, and the value of the minimum utility usage (Q_P^{min}) supplied to the first subnetwork is equal to 15,260 kW. We assume that the data in Table 1 correspond to a process in operation whose utility needs are provided by a steam-power facility described in Fig. 2, where other process steam demands are included. By solving the steam balance, one can find the HP-steam flow required to meet all plant demands. Figure 3(a) indicates that 162.18 ton/h are needed.

High-, medium- and low-pressure steam, however, are available in the steam-power plant. Using the transportation formulation introduced by Cerdá *et al.* [12], the minimum utility cost can be calculated. First, we need to assign relative costs to utilities, with these

Table 1. Process stream data

Process Stream	F_iCp_i(Kw/°K)	T_{in}(°K)	T_{out}(°K)	Q_i(Kw)
C_1	25	301	538	5925
C_2	100	335	480	14500
C_3	115	305	415	12650
C_4	Fλ = 2333	385	385	2333
H_1	106	450	392	-6148
H_2	125	432	320	-14000
Available Utilities:	HP Steam (550°K) MP Steam (508°K) LP Steam (401°K)			

Table 2. Evaluation of the net heat demand at each problem subnetwork ($\Delta T_m = 10°C$)

Sub-Network	Temp. Interval (°K)	Process Stream	Q_i (KW)	Net Heat Demand (KW)
1	>498	C_1	1000	1000
2	440-498	C_1 C_2	1450 4000	5450
3	422-440	C_1 C_2 H_1	450 1800 -1908	342
4	391-422	C_1 C_2 C_3 H_1 H_2	775 3100 2760 -3286 -3875	-526
5	385-391	C_1 C_2 C_3 C_4 H_1 H_2	150 600 690 2333 -636 -750	2387
6	335-385	C_1 C_2 C_3 H_1 H_2	1250 5000 5750 -318 -6250	5432
7	301-335	C_1 C_3 H_2	850 3450 -3125	1175

relative costs reflecting the pressure level. The heat flow cascade depicted in Fig. 3(b) shows two utility pinches, one for each utility past the first [13]. The overall utility usage in Fig. 3(a) and (b) is the same, but in the latter the temperature level (quality) of utility requirements has been *minimized*. A change in the utility allocation policy has occurred, and this produces a reduction in the value of the HP-steam flow required to meet all plant demands. It drops from 162.18 to 138.35 ton/h, both values computed by solving the steam balance. Between these two extreme utility allocations there are many others also reaching the minimum utility usage target (Q_P^{min}). Using them, HP-steam needs would be in between.

A heuristic commonly used by process engineers suggests fulfilling each heat demand with the cheapest steam source available. Frequently, it provides a good solution, i.e. a low boiler steam flow. When heat integration is considered, however, the power-generating structure becomes appreciably nonoptimal, and such a heuristic utility allocation could lead to rather large steam letdown flows.

4. ECONOMIC HEAT RECOVERY FOR EXISTING PLANTS

A better way to cut fuel expenses down is simply to reduce the current heat load Q_P by installing heat recovery networks at some processing systems. In doing so, the boiler load required to meet every process need can be greatly diminished.

In their review of process synthesis, Nishida *et al.* [2] partitioned the heat exchanger network problem into three major steps, the first being the preanalysis for setting targets. The authors also indicated that an economic network usually requires the least number of utilities (Q_P^{min}) and comprises the least number of heat exchangers. The optimal network design is therefore one that minimizes Q_P. Systematic procedures have already been proposed to determine Q_P^{min} [3,12]. The target Q_P^{min} comes from the assumption that any reduction in heat load causes a decline in both the boiler steam load and the fuel consumption.

Indeed, the real process heat recovery target is the lowest boiler fuel consumption. Since steam is used both for generating power and supplying heat, the new goal implies the use of a procedure that simultaneously considers process power and heat needs. In the case of an existing plant, the analysis should include the current power-generating structure. The newly defined target leads to the concept of limiting or economic utility usage Q_P^{lim}, which is the maximum heating utility usage that can still be satisfied through a boiler load that is as small as possible. A related notion is given by the difference $\Delta Q_{econ}^R = Q_P - Q_P^{lim}$, where ΔQ_{econ}^R stands for the economic heat recovery, i.e. the minimum heat recovery that still produces the highest energy savings.

The previous target Q_P^{min} is the minimum value Q_P^{lim} can take. Sometimes they are equal. For new plants they surely are. For existing plants usually $Q_P^{lim} > Q_P^{min}$. The gap between them becomes particularly large when a significant fraction of Q_P is fulfilled with low- and medium-pressure steam. In such cases a further decrease of Q_P below Q_P^{lim} through heat recovery does not yield any boiler steam savings. In other words, lowering Q_P beyond Q_P^{lim} implies investiments in heat transfer equipments to get no fuel savings.

This paper does not intend to prove that $Q_P^{lim} \geqslant Q_P^{min}$. It only proposes the least boiler load as the target for the HX-network and provides a systematic approach toward finding the "minimum" heat recovery ΔQ_{econ}^R requred to reach the target if an optimal utility allocation is made. By applying the procedure to update an existing plant, Q_P^{lim} can be obtained and compared with Q_P^{min}. The next section looks for a physical explanation of the difference between Q_P^{lim} and Q_P^{min}.

5. ECONOMIC SHIFT FROM PINCH CONDITIONS

An energy balance for the steam system involving every steam, power and heat demand to be met in a chemical complex can show the importance of consid-

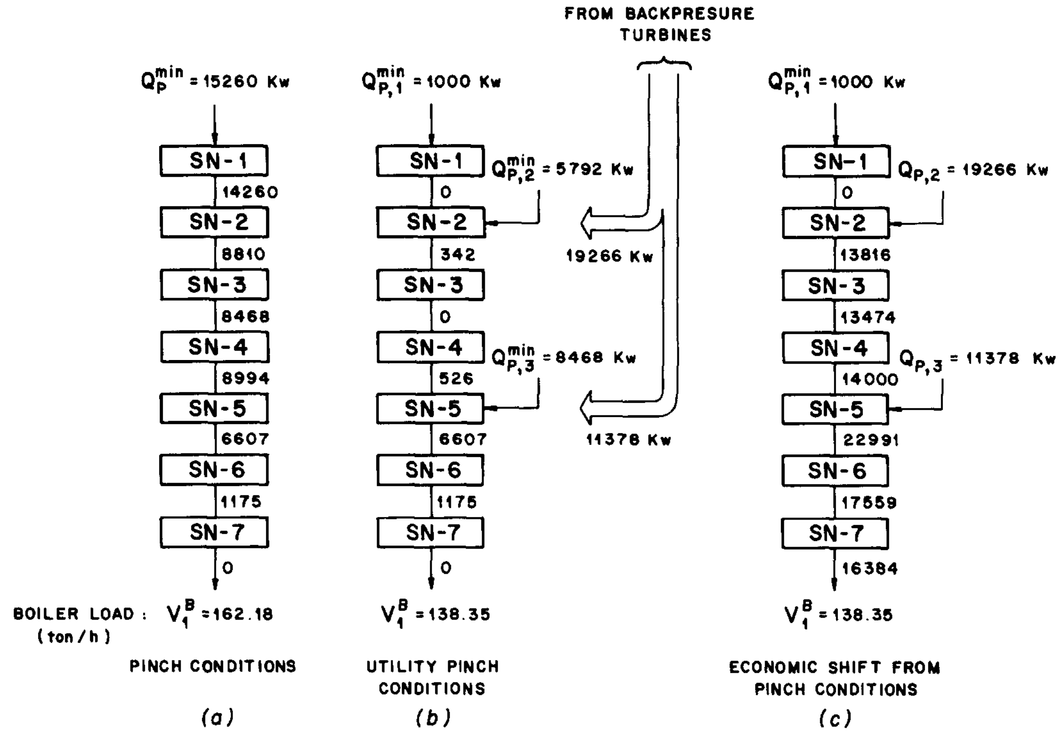

Fig. 3. Economic shift from pinch point conditions.

ering the present power-generating system structure in the analysis:

$$\sum_{k=1}^{N} V_k^B(H_k^B - h_D) = W + Q_P + \sum_{k=1}^{N} S_k(H_k - h_W)$$

$$+ Q_L + Q_E + Q_D - \sum_{k=1}^{N} V_k^{RB}(H_k^{RB} - h_D). \qquad (1)$$

Equation (1), which represents the energy balance over the steam distribution system and the boiler water makeup deaerator, shows that the energy flow provided by burning fuel serves to satisfy process heat load Q_P, power load W, process steam demand S and heat losses Q_L. Both pump work to drive condensates and steam vents are neglected. The last term in the RHS indicates the energy savings due to the heat flow recirculated in the heat recovery boilers. As usually happens in a minimization problem where certain requirements should be met at minimum cost, steam production and demand are not necessarily fully balanced. Then, a slack variable Q_E is introduced to stand for the heat taken by cooling water from both the condensing turbine exhausting steam and the LP-steam oversupply V_E for which no plant demand remains to assign. The value of Q_E is a measure of the steam Rankine-cycle overall efficiency η:

$$\eta = 1 - \frac{Q_E}{\sum_k V_k^B(H_k^B - h_D)};$$

$\eta = 1$ when Q_E is zero, and its value drops as Q_E goes up. Obviously, the existence of a condensing turbine in the power-generating system prevents η from being equal to 1. However, when power needs supplied by the steam turbine system largely exceed other demands, the use of a condensing turbine is economically attractive in spite of the lowering of η below 1, unless gas turbines are considered.

In Eqn (1), the RHS includes all process needs to be met through the steam Rankine cycle. To lower boiler fuel consumption, one should look for cost-effective process design changes. In particular, interest here is focused on the process heat load and the effect of its reduction over the heat flow supplied by burning fuel at boilers, i.e. the LHS of Eqn (1).

Let ΔH_C be the overall heat flow required by cold streams according to process specifications shown in Table 1. Let ΔQ^R be the actual heat flow recirculated into the process. Then, Eqn (1) can be written as follows:

$$\sum_{k=1}^{N} V_k^B(H_k^B - h_D) = W + \Delta H_C$$

$$+ \sum_{k=1}^{N} S_k(H_k - h_W) + Q_L + Q_E + Q_D \qquad (1')$$

$$- \left\{ \sum_{k=1}^{N} V_k^{RB}(H^{RB}{}_k - h_D) + \Delta Q^R \right\}$$

where

$$Q_P = \Delta H_C - \Delta Q^R \quad ,$$

$$Q_P^{min} \leqslant Q_P \leqslant \Delta H_C \quad .$$

Heat integration here is intended for an existing chemical process without changing the current steam-power-plant configuration. For the example problem introduced in the previous section, Table 1 gives the process specifications and Fig. 1 depicts the current steam-power facility. At present, no heat recovery is made at the process. Under such conditions, MP and LP-steam from backpressure turbines can provide heat in the amounts shown in Fig. 3(b). By comparing such values and the heat requirements at utility pinch conditions also shown in Fig. 3(b), it follows that the amount of heat available surpasses the minimum heat demand by as much as 16,384 kW. Recovering heat to reach pinch conditions, therefore, has no economic sense. There is no reason to lower process heat demands below the heat flow already available owing to power needs. If the plant is to be operated at minimum cost, one should limit the extent of process heat recovery so as to allocate all the available heat. In doing so, cooling water is to be used to remove 16,384 kW from process hot streams, and utility pinches no longer occur [see Fig. 3(c)]. In this case, Q_P^{lim} and ΔQ^R_{econ} are equal to 31,644 and 3764 kW, respectively.

Recycling process heat over ΔQ^R_{econ} implies recovering heat where the overall system is a heat source. It is like supplying heat in the source region to cause a similar increase in utility cooling [11]. The additional cooling needs arise at the LP-steam oversupply condenser. One can obtain a similar conclusion from Eqn (1′). Each RHS term standing for process needs W, ΔH_C and S_k does not depend on the extent of heat recovery ΔQ^R or on Q_D. Deaerator heat demand Q_D mostly arises from process steam requirements S_k. By neglecting Q_L, only Q_E can vary with ΔQ^k. Since Q_P^{lim} is the maximum utility usage still satisfied by a minimum boiler capacity, an increase of ΔQ^R over ΔQ^R_{econ} would leave LHS value of Eqn (1′) unchanged. Therefore, Q_E should grow by $\Delta Q^R - \Delta Q^R_{econ}$ to keep the balance between incoming and outgoing energy flows. Curves showing typical variations of both boiler fuel consumption and LP-steam oversupply V_E with ΔQ^R are included in Fig. 4.

At first sight, it seems appropriate to find ΔQ^R_{mx} by using previous procedures like Linnoff's problem table, but accounting for the additional steam flows. However, the amounts of saturated medium- and low-pressure steam available for heating use are not known data but steam-header temperature functions. In turn, such temperatures depend on the utility allocation policy and ΔQ^R, both being the values to optimize. Therefore, it becomes necessary to develop a new formulation that also provides the mathematical framework for further study of the driver allocation problem, as proposed by Nishio *et al.* [4]. In such a problem the power system structure also becomes a problem variable.

6. PROBLEM FORMULATION

6.1 Problem data

6.1.1 Steam distribution system specifications. An index k is introduced to identify each steam header, as well as the set of linked variables. It can vary from 1 to N going from the highest to the lowest pressure steam header; i.e. $P_{k+1} < P_k$. The vacuum header is considered as the $(N + 1)$th. header.

6.1.2 Power system structure. This information is provided through the values W_{jk} standing for the overall power supplied by the set of steam turbines whose inlet and outlet headers are j and k, respectively. Index j can vary between 1 and N, while k takes values between $J + 1$ and $N + 1$.

6.1.3 Inlet and outlet temperatures of process streams. Cold process stream temperatures define the problem temperature range. As already done by Linnhoff and Flower [3], the problem temperature range can be partitioned into a set of smaller intervals to bring about the concept of a temperature level at which heat is required by cold streams. From the utility viewpoint, it makes sense to establish as many tem-

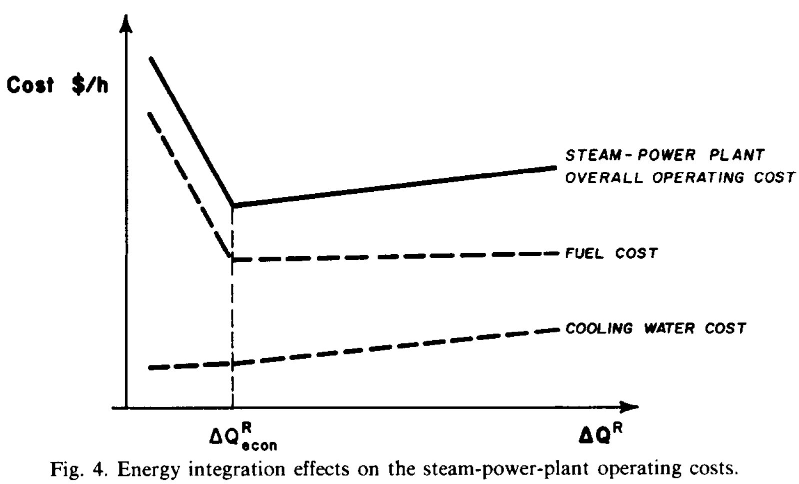

Fig. 4. Energy integration effects on the steam-power-plant operating costs.

perature intervals as the number of headers N in the steam system. For the configuration shown in Fig. 2, heat required by cold streams can be classified into high-, medium- and low-level demands (see Table 3). A high-level demand is one that can be met by using fired heaters or high-pressure steam. In turn, either fired heaters or high- or medium-pressure steam can be allocated to supply medium-level heat, and so forth. Temperature intervals characterizing each heating level are determined by taking into account (i) process flow diagrams; (ii) steam-header operating pressures P_k and (iii) the minimum allowed temperature difference in steam heaters, ΔT_m. Applying Linnhoff and Flower's procedure to the set of cold streams, one can find the value of the kth-level process heat load $\Delta H_{C,k}$. For the case illustrated in Fig. 2, the index k can vary from 1 to N, going from the highest to the lowest level heat demand.

6.1.4 Other steam user demands. The rate of steam delivered by the boilers should be high enough to meet other demands such as (i) process steam injection, denoted by S_k, which stands for the steam flow from the kth header to be used in the process; (ii) low-pressure steam supplied to the deaerator to heat up boiler feed water makeup V_D.

6.1.5 Steam flow rate raised in heat recovery boilers. It is assumed that heat recovery boilers have no supplementary firing. Otherwise, they can be treated as fired boilers with finite lower bounds on their productions. Let V_k^{RB} stand for the fixed steam flow supplied by the heat recovery boiler to the kth header.

6.2 Problem variables

Quantities that can be adjusted so as to minimize the *steam-power-plant fuel consumption are the problem variables*. They are

(1) heat load alloted to the steam header k (Q_k), $k = 1, 2, \ldots, N$,
(2) steam flow to letdown between headers k and $k + 1$, V_k^L.
(3) steam flow supplied to steam header k by fired boilers, V_k^B.

6.3 Problem objective function

When all fired boilers are operated at the same high pressure, the energy flow required to generate a boiler load V_1^B is given by

$$E = \frac{1}{\epsilon} V_1^B (H_1^B - h_D),$$

where H_1^B is the specific ethalpy for the superheated high-pressure steam supplied to the steam distribution system, h_D the boiler feed water enthalpy and ϵ the boiler efficiency. When the function $E = E(V_1^B)$ monotonically increases with V_1^B, the minimum fuel consumption is obtained through minimizing either the fuel E or boiler load V_1^B. It can be easily proved that such a situation arises when the condition (d ln ϵ/d ln V_1^B) < 1 holds, which is usually true for the entire boiler operating range [9]. If the steam system includes other generating units operating at lower pressures, the expression for E becomes

$$E = \sum_k \frac{1}{\epsilon_k} V_k^B (H_k^B - h_D) \tag{2}$$

where the enthalpy difference $H_k^B - h_D$ acts as a weighting coefficient favoring, when possible, the production of low-pressure steam. If $(\mathrm{d}E/\mathrm{d}V_k^B) > 0$, $k = 1, 2, \ldots, N$, any decrease in the overall boiler load leads to a reduction in the boiler fuel consumption.

Choosing the boiler load as the problem objective could lead to a mathematical program having many optimal solutions, some of which are of interest [see Fig. 3(b) and (c)]. The sought solutions are those featuring both the least boiler capacity and a maximum value for the steam Rankine-cycle efficiency. A top value for such an efficiency occurs when LP-steam oversupply V_E is minimized. By adding such a steam flow to the boiler load

$$\sum_{k=1}^{N} V_k^B (H_k^B - h_D) + V_E \lambda_N \quad , \tag{2'}$$

we define a problem objective function that only takes a minimum value when the economic heat flow ΔQ_{mx}^R is recovered. Besides, expression (2') is obviously an objective function for the utility allocation problem, when no heat integration is allowed. In this case, E given by (2) or its simpler expression V_1^B, when all fired boilers are operated at the same high pressure, are alternative objective functions.

6.4 Problem feasible region

The problem feasible region is defined by the following set of constraints.

(a) Mass-balance for each steam header:

$$V_k^B + V_k^{RB} + V_{k-1}^F + \sum_{j=1}^{k-1} V_{jk}^W + V_{k-1}^L$$

$$= \sum_{m=k+1}^{N+1} V_{km}^W + \frac{Q_k}{H_k - h_k} + V_k^L + S_k, \tag{3}$$

where the additional steam ΔV_k raised by desuperheating the heating steam flow from the kth header can be calculated from

$$\Delta V_k = \frac{H_k - H_k^{(s)}}{\lambda_k(\mathrm{H}_k - h_k)} Q_k. \tag{4}$$

Superior index (s) denotes saturated steam property. In Eq. (3), V_{jk}^W is the steam flow supplied from header j to the turbine set exhausting at P_k, and V_k^F is the flow rate of steam at P_k, generated by flashing a higher pressure condensate. A linear expression for V_k^F in terms of Q_k can be written

$$V_k^F = \alpha_k \left\{ \frac{Q_{k-1}}{H_k - h_k} + \sum_{j=1}^{k-2} \left(\frac{Q_j}{H_j - h_j} \right) \prod_{q=j}^{k-2} (1 - \alpha_q) \right\}, \ k = 2, 3, ..., N \quad (5)$$

where

$$\alpha_q = \frac{h_q - h_{q+1}}{\lambda_{q+1}}.$$

(b) Energy balance for each steam header:

$$V_k^B H_k^B + V_k^{RB} H_k^{RB} + V_{k-1}^F H_k^{(s)} + \sum_{j=1}^{k-1} \Gamma_{jk}^W + \Gamma_{k-1}^L = \sum_{m=k+1}^{N+1} (W_{km} + \Gamma_{km}^W) + \frac{Q_k H_k}{H_k - h_k} + \Gamma_k^L + S_k H_k, \quad (6)$$

where Γ_{jk}^W standing for the enthalpy flow leaving the turbine set that operates between steam headers j and k is given by

$$\Gamma_{jk}^W = V_{jk}^W H_j - W_{jk}. \quad (7)$$

In turn, Γ_k^L is the letdown flow coming from kth header. Assuming isoenthalpic steam expansion, the value of Γ_k^L can be expressed as

$$\Gamma_k^L = V_k^L H_k. \quad (8)$$

(c) Nonisoentropic steam turbine expansion:

$$W_{jk} = \eta_{jk} V_{jk}^W [\mathrm{H}_j - f_k(H_j)], \quad (9)$$

where η_{jk} is the mechanical efficiency for the set of steam turbines working between headers j and k, and $f_k(H_j)$ is the exhausting steam enthalpy for such a turbine set when the expansion is isoentropic.

(d) Fulfillment of process heat load Q_P: When all process heat demands are satisfied through utility heating, the value of Q_P can be computed from process specifications. For the example problem of Table 1 high-, medium- and low-level heat requirements $Q_{P,k}$ are shown in Table 3. If $\Delta H_{C,k}$, $k = 1, 2, 3$ stand for such known values of $Q_{P,k}$, then the utility allocation feasible space is given by the following set of constraints:

$$Q_3 \leqslant \Delta H_{C,3},$$

$$Q_2 + Q_3 \leqslant \Delta H_{C,2} + \Delta H_{C,3},$$

$$Q_1 + Q_2 + Q_3 = \Delta H_{C,1} + \Delta H_{C,2} + \Delta H_{C,3} \ .$$

For an N steam-header system, it can be written:

$$\sum_{i=k}^{N} Q_i \leqslant \sum_{i=k}^{N} \Delta H_{C,i}, \ k = 2, ..., N;$$

$$\sum_{k=1}^{N} Q_k = \sum_{k=1}^{N} \Delta H_{C,k} \ .$$

It is important to differentiate $Q_{P,k}$ from Q_k. $Q_{P,k}$ is the process heat load of level k shown in Table 3, whereas Q_k is the heat alloted to steam header k to be found by solving the proposed mathematical formulation.

When heat integration is considered, process heat demands to be met through utility heating are diminished. In order to define the new utility allocation feasible space, one should first establish lower bounds for high-, medium- and low-level heat demands. They are called $Q_{P,k}^{min}$, and their values are obtained by solving the minimum utility cost problem. For the example problem, values for $Q_{P,k}^{min}$ are depicted in Fig. 3(b). Since a feasible heat integration scheme should be in between the two extreme situations, i.e. no heat recovery and maximum heat integration as given by $\{Q_{P,k}^{min}\}$, then the following sets of constraints are to be satisfied:

$$\sum_{i=1}^{k} Q_i \geqslant \sum_{i=1}^{k} Q_{P,i}^{min}, \ k = 1, 2, ..., N;$$
$$\sum_{i=k}^{N} Q_i \leqslant \sum_{i=k}^{N} \Delta H_{C,i}, \ k = 1, 2, ..., N. \quad (10)$$

(e) nonnegativity condition for each problem variable.

6.5 *The mathematical model*

The utility allocation and the economic heat recovery problems are described by a nonlinear mathematical programming problem with a linear objective function, which can be solved by using a minimization algorithm for constrained problems like the Successive Linear Programming method (SLP). Through a mathematical proof it will be shown that the algorithm converges to a problem stationary point.

However, although both the economic heat recovery and the utility allocation problems share nearly the same mathematical formulation, there is an essential feature that differentiates one from the other. When heat integration is considered, process heat loads at distinct temperature levels $\{Q_{P,k}\}$ are constrained variables rather than known values. This implies that the economic heat recovery problem (EHRP) is a mathematical relaxation of the utility allocation problem (UAP) because it involves additional degrees of freedom. Thus, the boiler load optimal value will now be lower than the one for the UAP problem by an amount as large as the actual chances to recycle process heat economically. It is important to remark that $\{Q_{P,k}\}$ are constrained variables. Otherwise, their values at the optimum will become zero.

When heat integration is considered, the smallest boiler capacity is surely obtained if both the amount and quality of the process heat load are minimized. This requires an appropriate use of the heat available at processing systems that avoid any available energy degradation. Here, lower quality means a greater contribution of low-level heat demands. The minimum utility usage in both quantity and quality can be found by solving the utility cost problem defined by Cerdá et al. [12]. All the utilities available at the chemical plant are ranked and assigned costs in the problem so that the optimal solution tends to use cheaper utilities as much as possible. Let $Q_{P,k}^{\min}$ stand for the kth-level heat load at the minimum-cost solution. Whatever the heat exchanger network design, the conditions established by constraints (1) should always be met. Although the minimum utility cost solution is not always the actual target to be achieved by the heat exchanger network to be designed, it still provides the necessary information $\{Q_{P,k}^{\min}\}$ for determining the economic utility usage $Q_P^{\lim}$ $(= \Sigma\, Q_k^*)$.

Although the process heat loads at different temperature levels $\{Q_{P,k}\}$ do not explicitly appear in Eqns (10), they are problem variables in the sense that there are many sets of values $\{Q_{P,k}\}$ that require the same type and number of utilities, represented by $\{Q_k\}$. From all the $\{Q_{P,k}\}$ corresponding to the optimal solution $\{Q_k^*\}$, the one leading to a heat exchanger network design with a minimum number of units is sought.

6.6 The solution procedure

As frequently recommended for nonlinear optimization problems involving a few nonlinear constraints in addition to linear ones, the SLP method has been to find a problem stationary point [5]. This method approximates the problem mathematical formulation (for UAP or EHRP) given by the nonlinear programming problem (NLP)

$$\begin{aligned} &\text{minimize } \mathbf{c}^T\mathbf{x} \\ &\text{subject to } A\mathbf{x} - \mathbf{b} \geqslant \mathbf{0}\ , \\ &\qquad \mathbf{g}(\mathbf{x}) = \mathbf{0}\ , \\ &\qquad \mathbf{x} \geqslant \mathbf{0}\ , \end{aligned}$$

through a linear programming problem (LP$_i$) in which each nonlinear function appearing in (NLP) is approximated by a first-order Taylor expansion:

$$\begin{aligned} &\text{minimize } \mathbf{c}^T\mathbf{x}, \\ &\text{subject to } A\mathbf{x} - \mathbf{b} \geqslant \mathbf{0}, \\ &\qquad \mathbf{g}(\mathbf{x}_i) + \Delta\mathbf{g}^T(\mathbf{x}_i)\,(\mathbf{x} - \mathbf{x}_i) = \mathbf{0}, \\ &\qquad \mathbf{x} \geqslant \mathbf{0}\ , \end{aligned}$$

where $\mathbf{c}^T\mathbf{x}$ represents either (2) or (2′), depending on the problem. The linear constraint set $A\mathbf{x} - \mathbf{b} \geqslant \mathbf{0}$ involves the restrictions defining the utility allocation feasible space. In turn, material and energy balances for each steam header and the expressions for Γ_k^L and Γ_{jk} are included in the nonlinear constraint set $\mathbf{g}(\mathbf{x}) = \mathbf{0}$.

The algorithm consists of an iterative scheme where at each step i the corresponding linear programming problem (LP$_i$) is solved. The optimal solution to (LP$_i$), $\mathbf{x}_i$, is used as the new approximating point, and all nonlinear functions are linearized around $\mathbf{x}_i$ in the iteration $(i + 1)$. A point $\mathbf{x}_i$ is said to be optimal for (NLP) if it is feasible within some tolerance and a certain convergence criterion on the succesive values of $f(\mathbf{x}_i) = \mathbf{c}^T\mathbf{x}_i$ or $\mathbf{x}_i$ is met [14].

To start the solution method, an initial point x_0, which can be infeasible, is needed. Its choice should be made so as to increase chances to get the problem global optimum. Looking at the mathematical model, it can be concluded that problem nonlinearities will all disappear if steam-header enthalpy values are known beforehand. In such case, Eqns (9) give the values for the variables V_{ij}^W. Moreover, consideration of steam-header energy balances becomes meaningless and the problem constraint set reduces itself to the steam-header material balances and the process heat load fulfillment restrictions, both given in this case by linear equations. Therefore, a proper choice of H_k values and the solution of the resulting low-size LP program by using the simplex method will provide a good initial, but still infeasible, solution.

Making a good choice of H_k values first requires figuring out the prevailing energy supply system operating conditions at the optimum. In the UAP problem, the only means of cutting off boiler load is to improve the steam Rankine-cycle efficiency by decreasing letdown flows V_k^L and, in particular, making the low-pressure steam oversupply V_E equal to zero. At the global optimum, therefore, letdown flows are generally quite low, and their effects over the steam-header enthalpy values can be neglected. On that basis, a good H_k-value selecting procedure for the UAP problem is proposed, where the enthalpy values are sequentially evaluated by going from the high-pressure to the low-pressure steam header after assuming that

(i) the temperature at the high-pressure steam header is approximately equal to that of the superheated steam supplied by the high-pressure boilers;
(ii) steam letdowns and low-pressure steam oversupply can be neglected.

When the EHRP problem is considered, an additional assumption is made:

(iii) $Q_k = Q_{P,k}^{\min}$, $k = 1, 2, \ldots, N$.

6.7 Convergence to a Kuhn–Tucker point

As seen before, the solution of the nonlinear mathematical program describing the NLP problem is found by solving a sequence of linear programming problems LP$_i$. It can be shown that the limit of the

LP_i optimal solution sequence $\{\mathbf{x}_i\}$ is an NLP-stationary point provided that

(1) each (LP_i) has at least a feasible solution,
(2) the sequence $\{\mathbf{x}_i\}$ converges.

The proof of this statement is included in the Appendix.

7. OPTIMAL NECESSARY CONDITION

As already considered, a further reduction in steam needs can be achieved through an improvement in the efficiency of the power- and steam-generating cycle. In doing so, low-pressure steam excess V_E that finds no use in the plant and requires utility cooling is minimized. Such optimal operating conditions always involve either no LP-steam oversupply or the closing of at least one of the letdown valves, as proved in the following theorem:

THEOREM

Consider either the UAP or the EHRP problem. Let $\mathbf{x} = (V_1^B, V_1^L, Q_1, V_2^L, \ldots)$ be a feasible solution. If $\mathbf{x}$ is the local optimal solution to the problem, then at least one of the letdown flows V_k^L, $k = 1, 2, \ldots, N - 1$ or the steam excess $V_E \equiv V_N^L$ in $\mathbf{x}$ is equal to zero.

Proof. By contradiction, suppose that all the letdown flows and $V_E = V_N^L$ are positive:

$$V_k^L > 0,\ k = 1, 2, \ldots, N.$$

By subtracting a very small quantiy δ from each letdown flow,

$$\overline{V}_k^L = V_k^L - \delta,\ k = 1, 2, \ldots, N,$$

where

$$\delta \leqslant \min(V_k^L),$$

all steam-header material balances are still satisfied except for $k = 1$. Setting up the mass balance at the high-pressure header by taking

$$\overline{V}_1^B = V_1^B - \delta < V_1^B,$$

a new improved feasible solution has been obtained with the other V_k^B remaining unchanged. This contradicts the assumption that $\mathbf{x}$ is a local optimum, and, therefore, at least one of the letdown flows or the LP-steam excess is null.

8. RESULTS

8.1 UAP problem

The economic advantages that bring about the optimal allocation of utilities in a chemical plant are first studied by applying the proposed optimization procedure to the example problem of Table 1. It is assumed that an energy conservation program should be undertaken in an existing large chemical plant to reduce fuel expenses by making cost-effective changes in design and operating conditions. Figure 2 shows the current structure and operating conditions of the plant steam-power-generating facilities. Process heat requirements are included in Table 1, where a list of process streams, their heat flow capacities and inlet/outlet temperatures are shown. So far, no effort has been made at the plant to recover energy available in the process streams. Since the steam distribution system comprises three headers, process heat needs have been partitioned into three temperature levels while considering both the steam saturation temperature at each header pressure and the minimum allowed temperature difference at steam heaters, ΔT_m. The set of temperature intervals defining the problem levels are displayed in Table 3, where ΔT_m is assumed to be equal to 10°C. Table 3 also includes the resulting partial and overall heat requirements at each temperature level, $\Delta H_{C,k}$, $k = 1, 2, 3$.

For the example problem, the mathematical representation introduced in this paper consists of 27 equality and inequality constraints given in terms of 25 variables. An optimal solution is found by sequentially solving the LP_i problems after around 21 iterations. The optimal values are shown in Fig. 5. The initialization step provides the following starting values for $\{Q_k\}$ = 6302, 9032 and 20073 kW in three iterations of the simplex algorithm. These starting values compare favorably with the optimal values, 4138, 11,196 and 20,073 kW.

In order to get the smallest boiler load, it follows that the optimal utility allocation policy causes all letdowns to decrease so much that both V_1^L and V_E become zero (see Fig. 5). This result is a clear sign that the steam Rankine-cycle efficiency has reached its maximum value. The common practice employed in the industry, i.e. to meet every process heat demand by using a utility of similar temperature level, would lead to the values shown at Fig. 6. These values obtained by solving the mathematical model after replacing restrictions (10) by

$$Q_k = \Delta H_{C,k},\ k = 1, 2, 3$$

Table 3. Evaluation of $\Delta H_{C,i}$, $i = 1, 2, 3$ for the example

Temp. Interval (°K)	Process Stream	Heat Demand (Kw)	Overall Heat Demand ($\Delta H_{C,k}$)	Lowest Utility Level Required
> 498	C_1	1000	1000	1 (HP Steam)
391-498	C_1 C_2 C_3	2675 8900 2760	14335	1 (MP Steam)
< 391	C_1 C_2 C_3 C_4	2250 5600 9890 2333	20073	3 (LP Steam)
		ΔH_C =	35408	

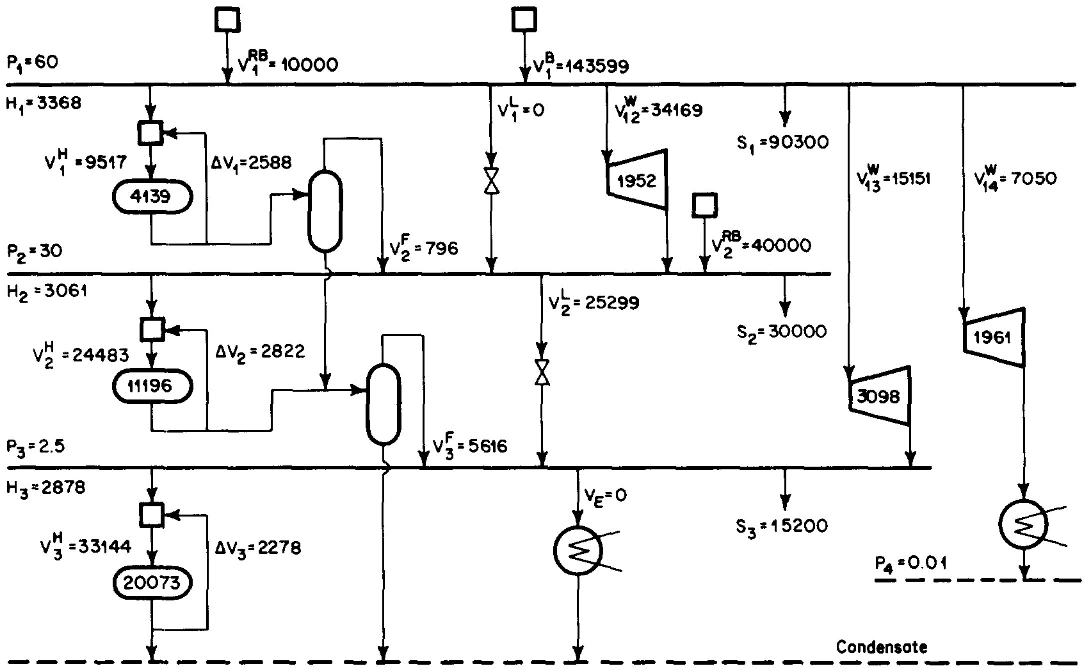

Fig. 5. Steam-power-plant operating conditions for an optimal heating utility allocation.

to impose the heuristic allocation policy. In this case, HP-steam requirements are slightly higher. Apart from confirming the rule as a good heuristic, the comparison between Figs 5 and 6 shows that the low-level heat load $\Delta H_{C,2}$ is met with medium- and high-pressure steam when the optimal utility allocation policy is adopted. Moving steam users upward in the steam system brings two major advantages: (i) The reduction of letdown flows yields a slight reduction in boiler load; (ii) the heat transfer area required at steam heaters is significantly decreased. However, differences between heuristic and optimal allocation policies could sometimes become more important.

The example problem has been solved for a steam

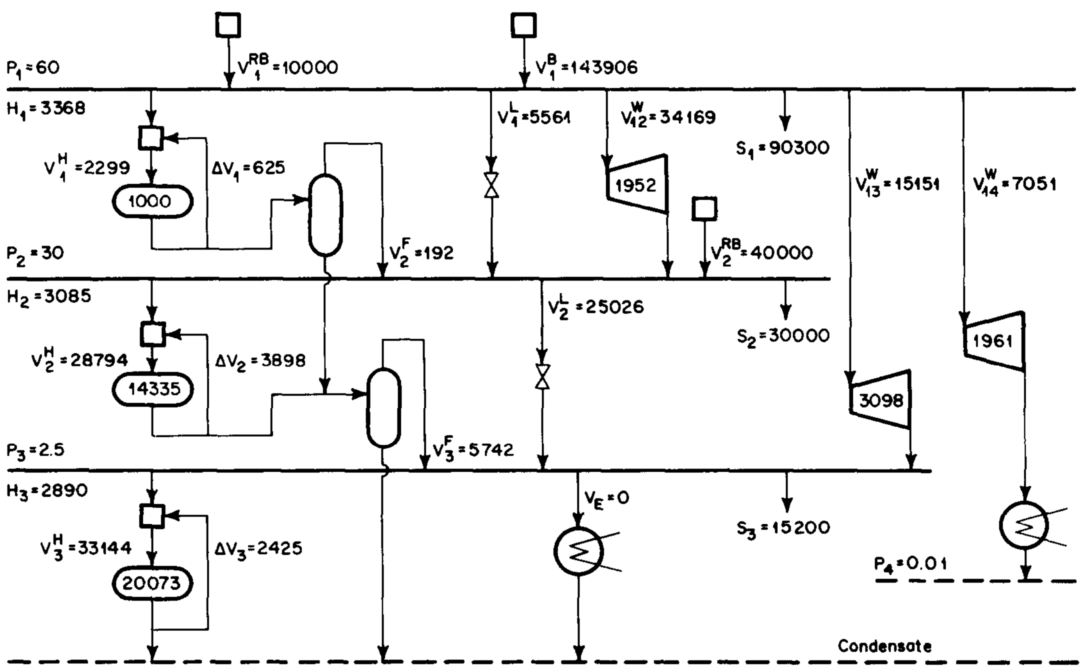

Fig. 6. Steam-power-plant operating conditions for the heuristic heating utility allocation.

system comprising only high-pressure fired boilers (see Fig. 5). From a mathematical viewpoint, the inclusion of medium- and/or low-pressure boilers means the introduction of new variables. Therefore, the effect on the overall fuel needs, if any, will be a decrease in their value, caused by a further reduction of steam letdown flows.

In any case, the optimal values for Q_k indicate the types and number of plant utilities to be consumed in an economic steam-heater network. The steam-heater network design problem is a particular case of the general network design problem where hot streams are the steam flows whose heat availabilities are given by the optimal $\{Q_k^*\}$. But many network configurations can achieve such a target. In agreement with what was found for the heat exchanger network problem, the lowest number of heaters can be proposed as another target for the sought design to further cut off fixed costs. The search for the optimal design based on the transportation procedure proposed by Cerdá and Westerberg [1] and later on improved by Isla and Cerdá [15] yields the heater network structure with a minimum number of units shown in Fig. 7.

It has been shown that the use of medium- and low-pressure boilers in the steam-generating system causes letdown flows to fall. Sometimes, however, the elimination of letdown flows becomes impossible for power-dominant plants. In such cases a steam driver reallocation is needed.

8.2 *Economic heat recovery problem*

Additional energy savings can be procured to a great extent by recovering the heat flow available in hot-process streams. Before synthesizing the heat recovery network, however, one should know the amount of each heating utility to allocate in the process so as to minimize fuel expenses. This information is obtained by solving the proposed nonlinear programming problem. First, all model data, including the lower bound for the process heat loads $\{Q_{P,k}^{min}\}$, are to be known. As stated before, the utility cost problem is used to evaluate $Q_{P,k}^{min}$ [12]. For the example problem, the transportation problem tableau is constructed by using the process heat availabilities and demand in each subnetwork listed in the Q_i column of Table 2. Since high-, medium- and low-pressure steam can act as heating sources, the tableau should include three additional source columns. Relative costs equal to 3, 2 and 1, respectively, are assigned to them. By applying the transportation algorithm, the following values are obtained: $\{Q_{P,k}^{min}\}$ = 1000, 5792 and 8468 kW and Q_P^{min} = 15,260 kW. They represent the required utility heating when the maximum process heat flow is recycled with a minimum energy degradation. Under these conditions, two utility pinches arise in the heat cascade flow [see Fig. 3(b)].

Now taking the values $Q_{P,k}^{min}$, k = 1, 2, 3, as the present process heat loads, the solution to the corresponding UAP problem surely provides the lowest boiler capacity value. This lower bound on the boiler load is obtained by assuming optimal process heat integration. The difference between the minimum boiler load (138.35 ton/h) and the one without heat recovery (143.6 ton/h) represents an upper bound for the steam savings. For the example problem, therefore, HP-steam and energy savings cannot be greater than 5.25 ton/h and 4310 kW, respectively. Figure 8 shows the steam-power-plant operating conditions. Note that a significant LP-steam oversupply V_E, as large as 23.9 ton/h, suggests the existence of a much cheaper alternative optimal solution. In other words, similar fuel savings can be achieved with lower investment.

The actual utility target for designing the heat recovery network of an existing plant is found by solving the EHRP problem with expression (2′) as the objective function. For the example problem, the initialization step provides initial values for the set $\{Q_k\}$ given by 1000, 16,991 and 10,540 kW in five iterations of

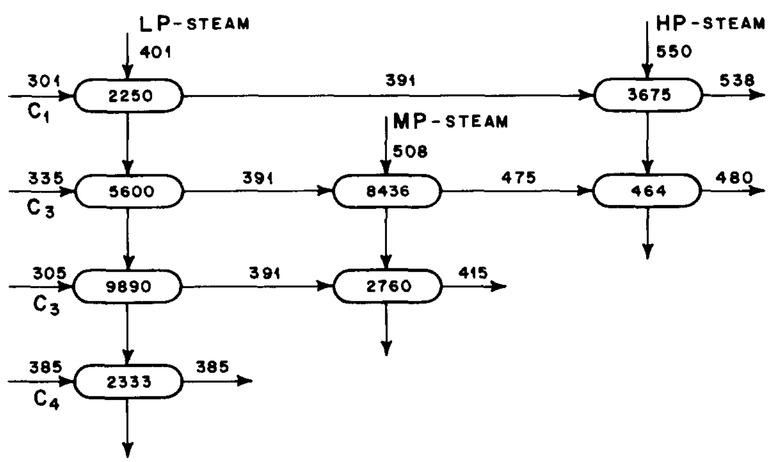

Fig. 7. Steam heaters network with a minimum number of heat exchangers corresponding to the optimal heating utility allocation.

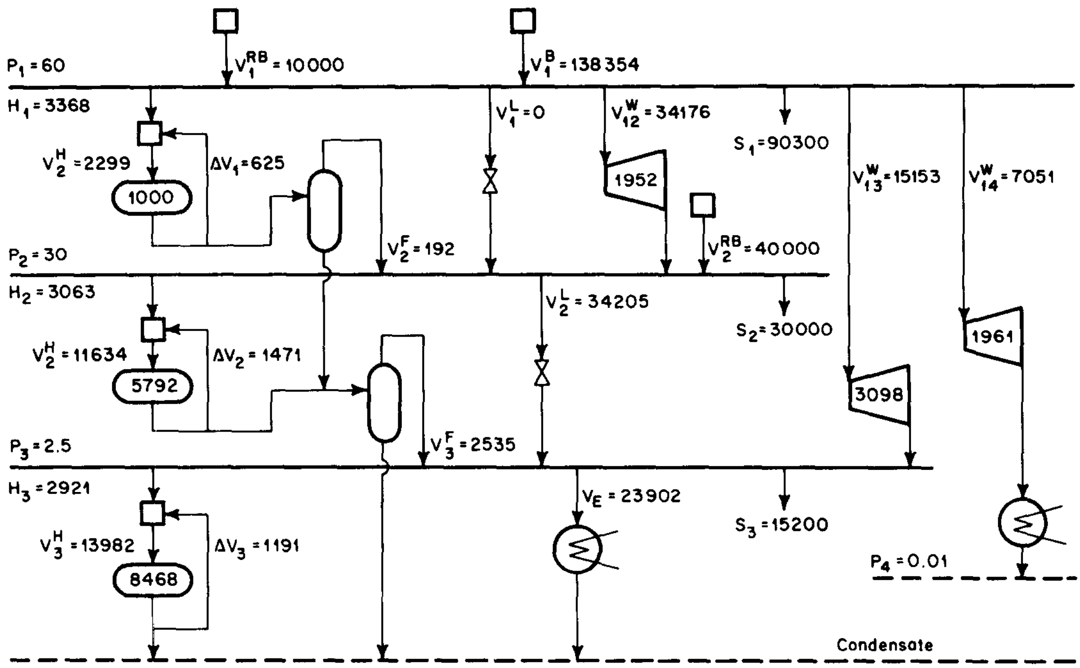

Fig. 8. Steam-power-plant operation at utility pinch conditions and $Q_k = Q_{P,k_{min}}$, $k = 1, 2, 3$.

the simplex method. The initial approximation for Q_P^{lim} is, therefore, equal to 28,531 kW–far above Q_P^{min} (=15,260 kW). Starting from this solution, the SLP algorithm obtains the optimal values for $\{Q_k\}$ in 20 iterations. By comparing $\{Q_k^*\} = 1000$, 19,266 and 11,378 kW with the initial approximations, it becomes clear that the initialization step makes the SLP algorithm quite efficient.

Figure 9 depicts the operating conditions at the steam-power facility when the process heat load is reduced through heat recovery from the current value (= 35,408 kW) up to Q_P^{lim} (= 31,644 kW). It is observed that the LP-steam excess V_E no longer occurs and that the steam letdown V_2^L has been sharply cut down from 34.2 to 10.6 ton/h. Moreover, the important difference between Q_P^{lim} (= 31,644 kW) and

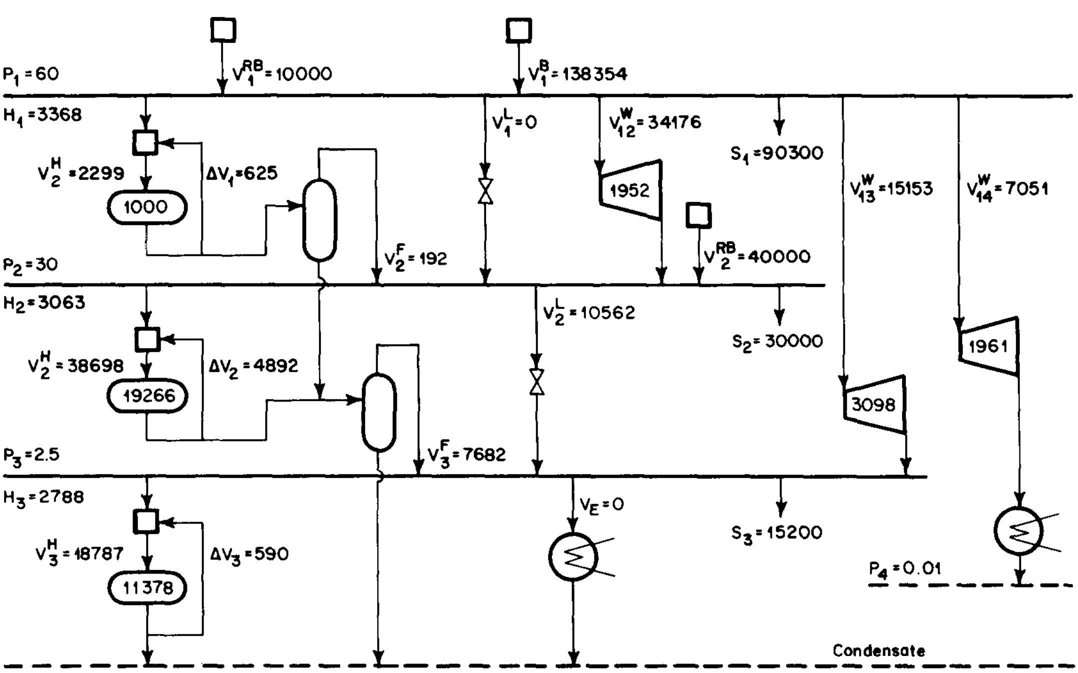

Fig. 9. Steam-power-plant operating conditions associated with the EHRP optimal solution.

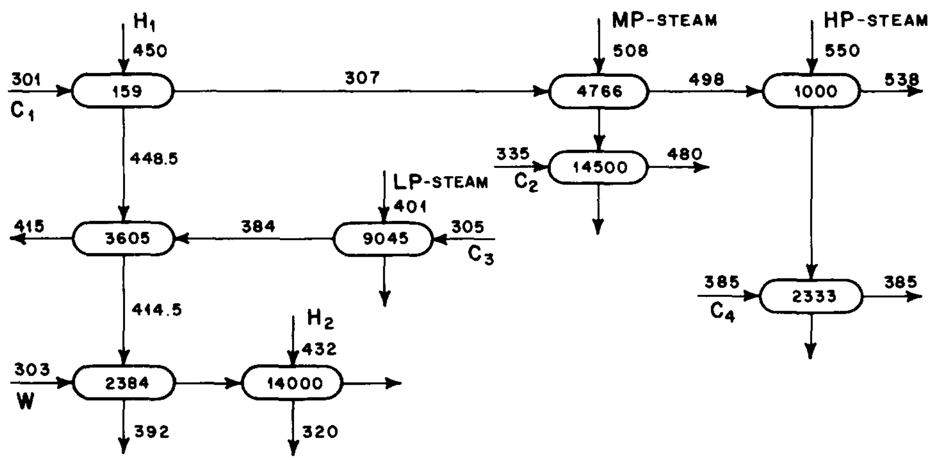

Fig. 10. Network with a minimum number of heat exchangers corresponding to the EHRP optimal solution.

Q_P^{min} (= 15,620 kW) implies that much less heat is to be recovered to get both the lowest fuel consumption and minimum investments.

By knowing the actual utility target (Q_P^{lim}), one can use any proposed synthesis technique [1,16] to find an optimal heat exchanger network design involving a low number of units.

With this purpose, the transportation synthesis technique is again applied to find the heat network design that gets the maximum energy savings at minimum fixed cost. Heat available from utility sources in the solution tableau are fixed at the Q_k^* values ($Q_1^* = 1000$; $Q_2^* = 19{,}266$; $Q_3^* = 11{,}378$) shown in Fig. 3 instead of at the previous targets, 1000, 5792 and 8468 kW, respectively.

The solution to the network design problem provides a heat integration scheme allowing process streams to reach specifications through utility heating in the amounts Q_k^*, $k = 1, 2, 3$ and a minimum number of heat exchangers. The optimal heat exchanger network involving the lowest number of units is displayed in Fig. 10.

Undoubtedly, the current configuration of the steam-power plant becomes nonoptimal when heat integration is considered. A measurement of the shift from the optimal configuration is given by the difference $Q_P^{lim} - Q_P^{min}$. Figure 11 shows the optimal solution to the EHRP problem if the structure of the steam-power system is simultaneously optimized. In doing so, HP-steam requirements are sharply reduced to 114.7 ton/h, yielding a steam savings of 23.65 ton/h. This value is an upper bound on the lost profits caused by the steam-power-plant bottlenecks.

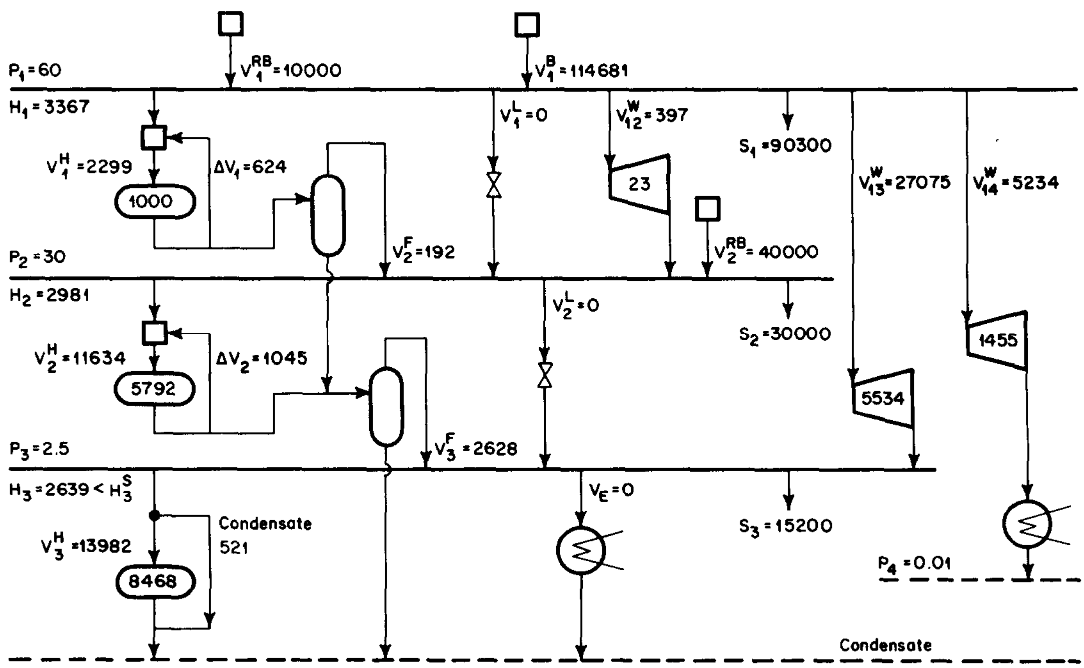

Fig. 11. Steam-power-plant optimal structure for the example problem.

NOMENCLATURE

E energy flow, kJ/s
H steam enthalpy, kJ/kg
h saturated liquid enthalpy, kJ/kg
N number of steam-headers, dimensionless
Q heat flow, kW
S process steam injection flow, kg/h
T temperature, K
V steam flow, kg/h
ΔV steam flow raise by desuperheating steam allotted to meet heating requirements, kg/h
ΔT_m minumum allowed temperature difference, K

Greek Symbols

ϵ boiler energy efficiency, dimensionless
η steam Rankine-cycle efficiency, dimensionless
Γ enthalpy flow, kJ/s
λ steam latent heat, kJ/kg

Superscripts

B fired-boilers
F flash or fuel
L letdown
lim limiting value
min minimum value
R heat recovery
RB heat recovery boilers
s saturated steam
W steam turbine

Subscripts

C cold stream
D deaerator
econ economic value
E steam excess
i header i or property of header i
j header j or property of header j
k header k or property of header k
L losses
mx maximum value
N header N
P process

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APPENDIX

Proof of the algorithm convergence to a K–T point

If $\mathbf{x}_i$ is the optimal solution for LP_i, then $\mathbf{x}_i$ satisfies the problem Kuhn-Tucker conditions:

$$\mathbf{c}^T - \mathbf{u}_i^{*T} A - \mathbf{v}_i^{*T} \nabla [\mathbf{g}(\mathbf{x}_{i-1}^*) + \nabla \mathbf{g}(\mathbf{x}^*_{i-1})(\mathbf{x}_i^* - \mathbf{x}_{i-1}^*)] \geqslant 0 \quad (A1)$$

$$\mathbf{x}_i^* \{\mathbf{c}^T - \mathbf{u}_i^{*T} A - \mathbf{v}_i^{*T} \nabla [\mathbf{g}(\mathbf{x}_{i-1}^*) + \nabla \mathbf{g}(\mathbf{x}_{i-1}^*)(\mathbf{x}_i^* - \mathbf{x}_{i-1}^*)]\} = 0 \quad (A2)$$

$$A\mathbf{x}_i^* - \mathbf{b} \geqslant 0 \quad (A3)$$

$$\mathbf{g}(\mathbf{x}_{i-1}^*) + \nabla \mathbf{g}(\mathbf{x}_{i-1}^*)(\mathbf{x}_i^* - \mathbf{x}_{i-1}^*) = 0 \quad (A4)$$

$$\mathbf{u}_i^* \geqslant 0 \quad (A5)$$

where $\mathbf{x}^*i-1$ is the optimal solution of (LP_{i-1}) and the vectors $\mathbf{u}^*i$, $\mathbf{v}^*i$ denotes the Lagrangian multipliers. Taking into account that $\mathbf{x}^*i-1$ is a constant vector for the problem (LP_i), the equations (A1) and (A2) are reduced to:

$$\mathbf{c}^T - \mathbf{u}_i^{*T} A - \mathbf{v}_i^{*T} \nabla \mathbf{g}(\mathbf{x}_{i-1}^*) \geqslant 0 \quad (A1')$$

$$\mathbf{x}^* \{\mathbf{c}^T - \mathbf{u}_i^{*T} A - \mathbf{v}_i^{*T} \nabla \mathbf{g}(\mathbf{x}_{i-1}^*)\} = 0 \quad (A2')$$

If the solution $\mathbf{x}^*i$ found satisfies the convergence criterion:

$$\|\mathbf{x}_i^* - \mathbf{x}_{i-1}^*\| < \epsilon$$

where ϵ is a positive enough small number, then:

$$\mathbf{x}_i^* \approx \mathbf{x}_{i-1}^*$$

Replacing this condition in (A1′), (A2′), (A3), (A4) and (A5) it is obtained:

$$\mathbf{c}^T - \mu^{*T} A^* - \lambda^{*T} \nabla \mathbf{g}(\mathbf{x}_i^*) \geqslant 0$$

$$\mathbf{x}_i^* \{\mathbf{c}^T - \mu^{*T} A - \lambda^{*T} \nabla \mathbf{g}(\mathbf{x}_i^*)\} = 0$$

$$A\mathbf{x}_i^* - \mathbf{b} = 0$$

$$\mathbf{g}(\mathbf{x}_i^*) = 0$$

$$\mathbf{u}^* \geqslant 0$$

that indicates that the limit of the sequence $\{\mathbf{x}_i\}$ is a Kuhn–Tucker point of the nonlinear mathematical program.